Maximum Turning Angle across an Oblique Shock

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THE geometry of an oblique shock is fixed by the density ratio across the shock, together with the invariance of the tangential component of velocity. It follows that, for a shock of fixed density ratio, the maximum turning angle θ_m and corresponding shock angle β_m are uniquely determined without placing any restrictions on the equation of state. The resulting algebraic expressions are simple enough to be useful.

The geometry of the oblique shock is shown in Fig. 1. By inspection, $w_1 = v \tan\beta$ and $w_2 = v \tan(\beta - \theta)$. Dividing the second relation into the first, and making use of $w_1/w_2 = \rho_2/\rho_1$ (by conservation of mass across the shock) yields

$$r = \tan\beta/\tan(\beta - \theta) \tag{1}$$

where $r \equiv \rho_2/\rho_1$ is the density ratio (for compression shocks, $r \geq 1$). Equation (1) appears in standard books on gasdynamics.[1,2] Trigonometric manipulation yields an explicit formula for $\theta(\beta,r)$,

$$\tan\theta = (r - 1)\tan\beta/(r + \tan^2\beta) \tag{2}$$

Consider now changing the shock angle β while holding r constant (this corresponds to keeping the shock strength fixed, or considering the same physical shock in different reference frames). From elementary calculus, the turning angle θ is an extremum when $d\theta/d\beta = 0$. Differentiating Eq. (2) with respect to β and setting $d\theta/d\beta = 0$ then yields

$$\tan\beta_m = r^{1/2} \tag{3}$$

which is the shock angle corresponding to maximum turning. The associated maximum turning angle θ_m is then found by substitution into Eq. (3),

$$\tan\theta_m = (r - 1)/2r^{1/2} \tag{4}$$

As a numerical example, consider a shock with $P_1 = 1$ atm and $P_2 = 50{,}000$ atm. In a perfect gas with $\gamma = 1.40$ the density ratio is the limiting value $r = 6$, yielding $\beta_m = 67.79°$, $\theta_m = 45.58°$. In water the density ratio (assuming an initial state near room temperature) is found to be $r \approx 1.51$, yielding $\beta_m = 50.8°$, $\theta_m = 11.7°$.

For certain problems, involving streaming flows in particular, it is physically more interesting to find the maximum turning angle for a fixed upstream Mach number M_1. This calculation is somewhat more complicated because it necessarily involves the fluid equation of state. Since r is uniquely fixed by the normal upstream Mach number $M_{1n} \equiv M_1 \sin\beta$ we can write

$$\theta = \theta[\beta, r(M_{1n} = M_1 \sin\beta)]$$

Following the rules of differentiation then gives

$$(\partial\theta/\partial\beta)_{M1} = (\partial\theta/\partial\beta)_r + (\partial\theta/\partial r)_\beta (dr/dM_{1n}) M_1 \cos\beta$$

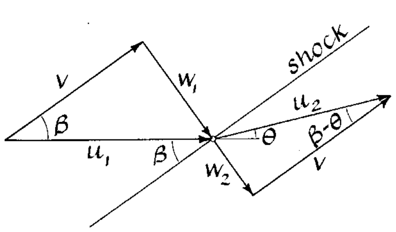

Fig. 1 Oblique shock geometry.

which is set equal to zero for an extremum. The partial derivatives on the right-hand side can be found from Eq. (2). Omitting the details, one then finds for the extremum

$$0 = r - \tan^2\beta + [M_{1n}/(r - 1)]dr/dM_{1n} \tag{5}$$

With $r = r(M_{1n} = M_1 \sin\beta)$, this is an implicit equation for the shock angle β_m' corresponding to maximum turning θ_m' at fixed M_1. In order to actually carry out the calculation, a particular relation $r(M_{1n})$ is needed: for example, a perfect gas has

$$r = (\gamma + 1)M_{1n}^2/[(\gamma - 1)M_{1n}^2 + 2]$$

It is still possible, however, to learn something about an arbitrary fluid. If the shock Hugoniot curve does not have a density minimum at finite pressure (see for example Hayes[3]), it follows that $dr/dM_{1n} \geq 0$ everywhere. Then Eq. (5) yields

$$\tan\beta_m' \geq r^{1/2} \tag{6}$$

where r is the density ratio at the extremum. With this result, Eq. (4) yields

$$\tan\theta_m' \leq (r - 1)/2r^{1/2} \tag{7}$$

If the shock Hugoniot does pass through a density minimum, the inequalities in Eqs. (6) and (7) would be reversed above that point.

References

[1] Shapiro, A. H., *The Dynamics and Thermodynamics of Compressible Fluid Flow*, Vol. 1, Ronald Press, New York, 1953, p. 535.

[2] Liepmann, H. W. and Roshko, A., *Elements of Gasdynamics*, Wiley, New York, 1957, p. 87.

[3] Hayes, W. D., "The Basic Theory of Gasdynamic Discontinuities," edited by H. W. Emmons, *Fundamentals of Gasdynamics*, Princeton University Press, Princeton, N. J., 1958, pp. 416–481.

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Pressure and Velocity Fields in the Sondhauss Oscillator

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Introduction

THIS article considers the situation of heat addition to a gas-filled cavity which, under certain conditions, results in resonant acoustic oscillations. The general configuration studied in this investigation was originally conceived by Sondhauss,[1] but received very little additional attention until a dissertation by Feldman.[2] Reasons for this current interest include the similarity of the Sondhauss oscillations to the heat generated gas oscillations occurring in rocket engine combustion chambers and gas furnaces. Additionally, it has been speculatively proposed to incorporate the Sondhauss phenomenon into a plasma oscillator for use as an alternating-current MHD power generator,[3] or it could serve as a high intensity sound source for environmental testing.[4]

Prior to any practical exploitation of the Sondhauss oscillator, it is essential that a thorough understanding of the phenomenon be obtained. Accordingly, this article presents a summary of an experimental investigation[5] directed pri-

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